

Evaluation of the maximized power of a regenerative endoreversible Stirling cycle using the thermodynamic analysis



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ABSTRACT

In this communication, the optimal power of an endoreversible Stirling cycle with perfect regeneration is investigated. In the endoreversible cycle, external heat transfer processes are irreversible. Optimal temperature of the heat source leading to a maximum power for the cycle is detained. Moreover, effect of design parameters of the Stirling engine on the maximized power of the engine and its corresponding thermal efficiency is studied.

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1. Introduction

Security of sustainable energy is one of the most important concerns for world societies [1]. A number of commercial enterprises are required for supplying climate-friendly energy technologies in future [2–5], including electricity produced from solar and wind energy [6–9], bio-fuels, and carbon dioxide (CO₂) capture and storage (CCS) [10–15]. In this regard, scrutinized planning with respect to the prospect plays an essential role toward better future. Greenhouse gases emission is one of the restrictions of utilizing fossil fuels which encourages development of higher efficiency and maximal electric [13–16] in energy production systems. Use of Stirling engines results in greenhouse gases emissions reduction into the atmosphere. The Stirling engine has a high potential to be applied for converting heat into the mechanical work with a high thermal efficiency. In theory its thermal efficiency might be as high as the Carnot efficiency when it is ideal regenerator, reversible, and having isothermal compression and expansion processes. Stirling engine is an external combustion engine, so it can be powered by various heat sources and waste heat [17–24]. Blank et al. [20] studied the power optimization of an endoreversible Stirling cycle and provided an estimate of potential performance for a real engine. Thombare and Verma [21] gathered the available technologies

and obtained achievements with regard to the analysis of Stirling engines and, at the end, presented some suggestions for their applications. Formosa and Despesse [23] conducted the modeling by means of the isotherm model in order to investigate the effects of dead volumes on the engine's output power and efficiency.

The Stirling engine cycle can be defined as a closed regenerative thermodynamic cycle consists of compression and expansion of the working fluid at different temperatures [25–35]. Tlili et al. [33] developed a first law model with additional consideration for an internal irreversibility parameter to examine the effects of irreversibility on net work, heat addition, and thermal efficiency. Their irreversibility parameter originates from the second law of thermodynamics for a real cycle. Their results regarding dead volume and regeneration effects support the observations of Kongtragool and Wongwises [35]. They also reported that larger irreversibilities in the Stirling cycle thermodynamic processes lead to smaller maximum network output and thermal efficiency. However, they did not quantify the roles of the heat exchangers, regenerator, and dead volume irreversibilities on the overall system irreversibility.

The enhancement of finite-time thermodynamics [36–40], lays on a useful tool for performance analysis of real engineering problems. The finite-time thermodynamic performance of the Stirling engine has been scrutinized by a great number of investigators [39–46]. The effect of heat transfer on the power output of a Stirling engine has been inquired by many researchers [4,23–30]. An optimal efficiency for maximum power output of a solar Stirling engine considering imperfect regeneration is achieved by Petrescu et al. [39,41]. Moreover, finite-time thermodynamics analysis of heat

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Nomenclature

A_H, A_L	heat transfer surface area between the endoreversible Stirling engine and either the high-temperature source or the low-temperature sink	T_2	temperature of the working fluid during the isothermal external heat rejection process
A_{reg}	heat transfer surface area of the Stirling engine regenerator	T_H, T_L	temperature of the heat source and the heat sink, respectively
C_v	constant volume specific heat	t_c	time required to accomplish the isothermal compression
Cl_i	decision index in TOPSIS method	t_h	time required for the heat transfer process from the regenerator to the working fluid
d	deviation index	t_R	time spent on the regenerative processes
d_{i+}	deviation or distance of i th solution from ideal solution	i	i th objective
d_{i-}	deviation or distance of i th solution from non-ideal solution	j	j th solution
m	mass of the gas	U_H, U_L	Overall heat transfer coefficient between the engine and either the low-temperature sink or the high-temperature source, respectively
P	power output	U_{reg}	overall heat transfer coefficient of the regenerator
Q_{1-2}	isothermal heat transfer from the engine to the low-temperature sink	X	vector of decision variables
Q_{2-3}	heat transfer from the regenerator to the working fluid		
Q_{3-4}	isothermal heat transfer into the engine from the high-temperature source		
Q_{4-1}	heat transfer captured by the regenerator from the working fluid		
R	gas constant		
T_1	temperature of the working fluid during the isothermal external heat input process		

Greek letter

λ	ratio of volume during the regenerative processes (volumetric ratio)
η	efficiency
μ	fuzzy membership function
δ	the Stefan's constant

engines is often limited to systems having linear heat transfer law which depends on the temperature differential both the reservoirs and engine working fluids [22,44–50]. Yagi et al. developed a mathematical model for the overall thermal efficiency of solar powered high temperature differential dish Stirling engine with finite heat transfer and irreversibility of regenerator and optimized the absorber temperature and corresponding thermal efficiency [42]. Tili investigated effects of regenerating effectiveness and heat capacitance rate of external fluids in heat source/sink at maximum power and efficiency [44]. Kaushik and Kumar studied the effects of irreversibilities of regeneration and heat transfer of heat/sink sources [46].

Solution of the multi-objective optimization problems is an extremely difficult goal which requires the simultaneous satisfaction of a number of different and even conflicting objectives. Evolutionary algorithms (EA) were initially extended and employed during the mid-eighties in an attempt to stochastically solve problems of this generic class [47]. A reasonable solution to a multi-objective problem is to investigate a set of solutions, each of which satisfies the objectives at an acceptable level without being dominated by any other solution [48]. Multi-objective optimization issues illustrate a possibly uncountable set of solutions which is named Pareto frontier, whose evaluated vectors depict the best possible trade-offs in the objective function space. In turn, multi-objective optimization of various thermodynamic and energy systems have been concentrated recently [49–56].

In real model on Stirling engines, heat regeneration is supposed to be considered imperfect. The optimal power theory design of the Stirling engine may be complex in this case. Assuming an ideal Stirling engine can lead to acceptable results in power output and thermal efficiency. This article concentrates on these elements using two scenarios.

2. Scenarios

2.1. Thermodynamic modeling

The regenerative endoreversible Stirling cycle is depicted in Figs. 1 and 2. Stirling cycle has two irreversible isothermal

processes and two reversible isobar processes. External heat transfer processes are carried out in finite time. Process 1–2 is performed at the constant temperature T_h and process 3–4 is performed at the constant temperature T_c .

In this work, it is assumed that a perfect regeneration is occurred in the regenerator.

2.1.1. Mathematical model

At a perfect regeneration, the amount of rejected heat from the hot working fluid to the regenerator mesh during process 1–4 is equal to the amount of absorbed heat by the cold working fluid

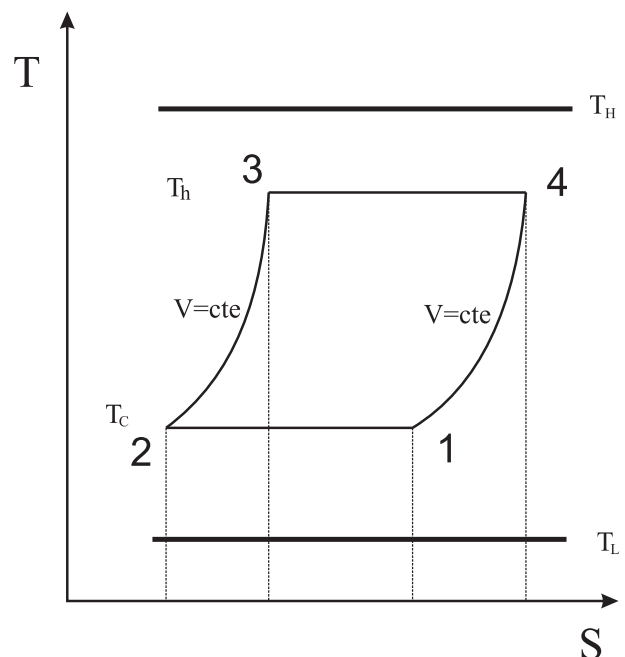


Fig. 1. Temperature-entropy diagram of the regenerative endoreversible Stirling engine.

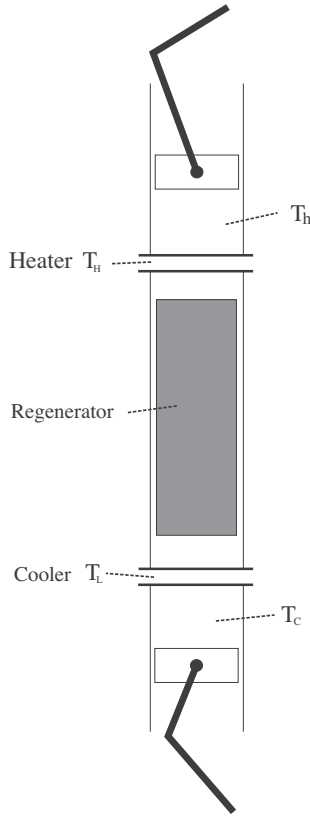


Fig. 2. Functional schematic of Stirling engine (note that the opposing pistons are 90 degrees out of phase).

during process 2–3. This assumption is acceptable, as regenerators with effectiveness of 98% and 99% have been reported [17,20]. Therefore, we have;

$$|Q_{2-3}| = |Q_{4-1}| \quad (1)$$

$$Q_H = U_H A_H (T_H - T_h) t_h = m R T_h \ln(\lambda) \quad (2)$$

$$Q_L = U_L A_L (T_c - T_L) t_c = m R T_c \ln(\lambda) \quad (3)$$

Thus, it can be shown that the net work of the cycle is obtained as follow [17,20,42–46];

$$W_{net} = m R (T_h - T_c) \ln(\lambda) \quad (4)$$

where m is the mass of the working fluid, R is the gas constant, T is the temperature of working fluid in each state of the cycle as described by subscripts and λ is the compression ratio. The output power is obtained by dividing the net work to the time period of the cycle [17,20,42–46];

$$P = \frac{W_{net}}{t} = \frac{m R (T_h - T_c) \ln(\lambda)}{t_h + t_c + t_r} \quad (5)$$

where t_r is the total time spent for the regeneration processes and t_h and t_c are the times spent for isothermal expansion and compression processes respectively. These time periods are defined as follows;

$$t_h = \frac{m R T_h \ln(\lambda)}{U_H A_H (T_H - T_h)} \quad (6)$$

$$t_c = \frac{m R T_c \ln(\lambda)}{U_L A_L (T_c - T_L)} \quad (7)$$

$$t_r = \frac{2 m c_v (T_h - T_c)}{U_{reg} A_{reg} (LMTD)_{reg}} \quad (8)$$

where U_H and A_H are the overall heat transfer coefficient and the heat transfer surface area for the hot side heat exchanger, respectively. Similarly, U_L , A_L and U_{reg} , A_{reg} , similar values for cold side heat exchanger and regenerator. T_H and T_L are the temperature of heat source and heat sink respectively. C_v is the specific heat at constant volume in $\text{kJ kg}^{-1} \text{K}^{-1}$ and is assumed to be constant.

Assuming the ideal regenerative, $(LMTD)_{reg} = 1 \text{ K}$. [4]. Substitution of Eqs. (6)–(8) into Eq. (5) leads to the following expression:

$$P = \left[\frac{T_h}{U_H A_H (T_H - T_h) (T_h - T_c)} + \frac{T_c}{U_H A_H \psi A_R (T_c - T_L) (T_h - T_c)} + \frac{2 C_v}{R U_{reg} A_{reg} \ln(\lambda)} \right]^{-1} \quad (9)$$

$$\psi = \frac{U_L}{U_H} \quad (9a)$$

$$A_R = \frac{A_L}{A_H} \quad (9b)$$

For simplicity, we considered the $x = \frac{T_c}{T_h}$ and $M = \frac{C_v}{R \ln \lambda}$, so Eq. (9) can be written as follow;

$$P = \left[\frac{1}{U_H A_H (T_H - T_h) (1 - x)} + \frac{x}{U_H A_H \psi A_R (x T_h - T_L) (1 - x)} + \frac{2M}{U_{reg} A_{reg}} \right]^{-1} \quad (10)$$

To determine the maximum power output, take the derivative of P with respect to T_1 and obtain the optimum temperature of working fluid at state 1;

$$\frac{\partial P}{\partial T_1} = 0$$

So;

$$T_{hopt} = \frac{\sqrt{\psi A_R T_L + x T_H}}{x (\sqrt{\psi A_R} + 1)} \quad (11)$$

Therefore, the thermal efficiency is calculated as follow;

$$\eta_t = \frac{P}{Q_H} \quad (12)$$

Substituting Eq. (11) into Eq. (10);

$$P_{max} = \left[\frac{1}{U_H A_H (T_H - T_{hopt}) (1 - x)} + \frac{x}{U_H A_H \psi A_R (x T_{hopt} - T_L) (1 - x)} + \frac{2M}{U_{reg} A_{reg}} \right]^{-1} \quad (13)$$

For the maximum power output it should be considered $U_H A_H = UA$ [4]. So Eq. (13) can be written as follow;

$$P_{max} = \left[\frac{1}{UA (T_H - T_{hopt}) (1 - x)} + \frac{x}{UA \psi A_R (x T_{hopt} - T_L) (1 - x)} + \frac{2M}{U_{reg} A_{reg}} \right]^{-1} \quad (14)$$

Also, the corresponding maximum net work is obtained as follow;

$$W_{net,opt} = m R T_{hopt} (1 - x) \ln(\lambda) \quad (15)$$

2.1.2. First scenario results

To have a numerical appreciation of the result, we consider the heat source temperature and cold source temperature 1300 K and 320 K respectively. Also $U_H A_H = U_L A_L = 2 \text{ kW K}^{-1}$, $x = 0.5$, $\lambda = 2$, $\psi = 1$, $A_R = 1$, and $U_{reg} A_{reg} = 1000 \text{ kW K}^{-1}$ [20]. We consider standard air as a working fluid with $R = 0.287 \text{ kJ K}^{-1} \text{kg}^{-1}$, $C_v = 0.718 \text{ kJ K}^{-1} \text{kg}^{-1}$ and $m = 9.3 \text{ g}$, as a total mass of the working fluid.

2.1.2.1. The effects of T_H . As be shown in Fig. 3, in various UA , by increasing T_H the output power increases. Also, in the certain T_H , by increasing UA , output power increases too. As is clear in Fig. 4, in various $(UA)_{reg}$, by increasing T_H the output power increases, also in certain T_H , with increasing $(UA)_{reg}$, output power is increased.

The effects of temperature of heat source on the output power at various values of x are presented in Fig. 5. At temperatures above 1100 K with increasing x , the power output is reduced. The slope of curve decreases by decreasing the difference between the temperature of working fluid in hot and cold spaces. According to Fig. 6, for different values of λ , with increasing T_H the output power is increased.

2.1.2.2. Effects of volumetric ratio (λ). Also in Fig. 7, the effects of volumetric ratio on the output power at various UA are presented that have similar behavior to Fig. 6.

Fig. 8 shows the effects of the volumetric ratio on the thermal efficiency at various values of temperature ratios, in which the efficiency increases by increasing λ and as the temperature difference is decreased the thermal efficiency is reduced. Fig. 9 shows the effects of volumetric ratio on the output power for different values of x that has similar behavior as the thermal efficiency be shown recently. Fig. 10 shows the effects of volumetric ratio on the output power for different values of $(UA)_{reg}$.

2.1.2.3. Effect of UA. By increasing the UA of heat exchangers the output power increases rapidly in comparison with volumetric ratio and heat source temperature that means the heat exchanger and regenerator with high effectiveness coefficient are more efficient, as be shown in Fig. 11.

2.2. Multi-objective optimization with evolutionary algorithms

2.2.1. General concepts of the multi-objective optimization

Imagine a decision-maker that would like to optimize objectives which are non-commensurable. The decision-maker does not know the priority of the objectives. Moreover, all objectives are of the minimization type. For a minimization kind of objectives, it can be changed to the maximization type by multiplying negative one. The procedure of a minimization multi-objective decision problem with objectives is represented as follows:

Given a non-dimensional decision variable vector $x = \{x_1, \dots, x_n\}$ in the solution space X find a vector x^* that minimizes a given set of K objective functions $f(x) = \{f_1(x^*), \dots, f_k(x^*)\}$. The solution space X is generally restricted by a series of constraints, such as $g_j(x^*) = b_j$ for $j = 1, \dots, m$, and bounds on the decision variables [48]. Overall, no

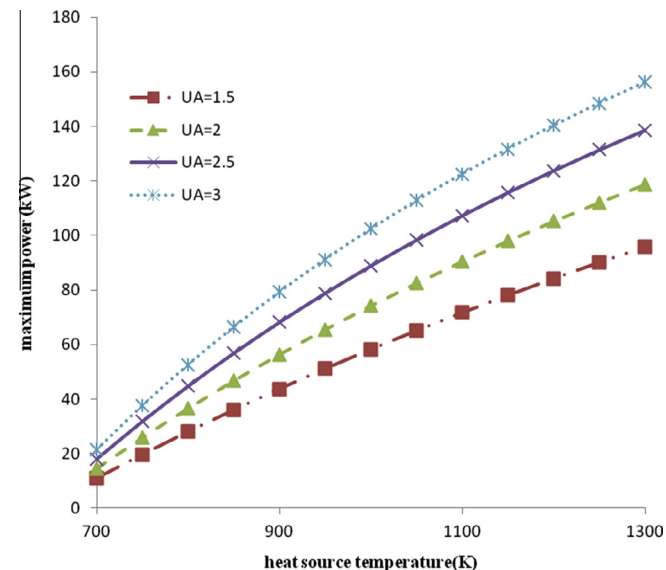


Fig. 3. Variation of the power of the Stirling engine for different the heat source temperature and UA.

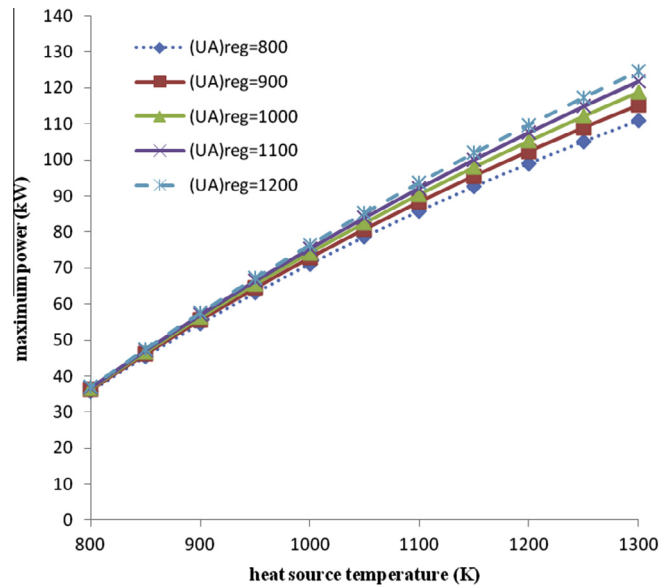


Fig. 4. Variation of the power of the Stirling engine for different the heat source temperature and $(UA)_{reg}$.

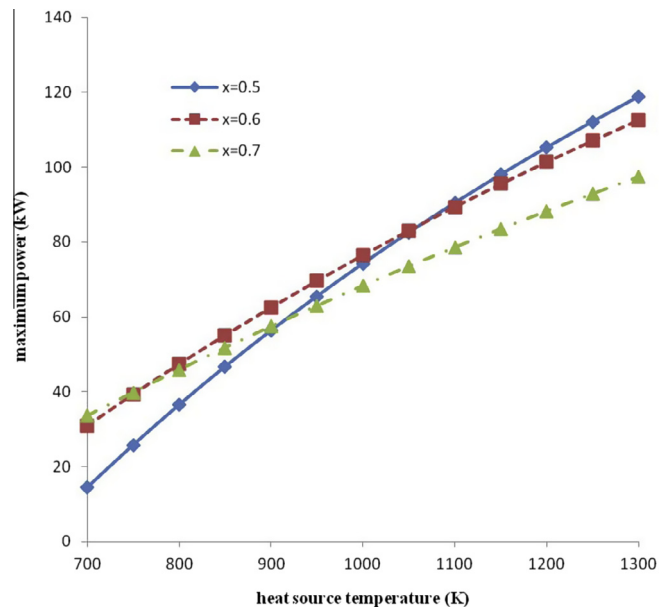


Fig. 5. Variation of the power of the Stirling engine for different the heat source temperature and x .

solution vector X exists that minimizes all the K objective functions simultaneously. In other words, in various real-life problems, objectives at issues conflict each other. Thus, optimizing X with respect to a uni-objective often leads to unacceptable results with respect to the other objectives. Thus, a new concept, well-known as the “Pareto optimum solution”, is used in multi-objective optimization problems. A feasible solution X is called “Pareto optimal” if there exists no other feasible solution Y that dominates solution X . By definition, a feasible solution Y is said to dominate another feasible solution X , if and only if, $f_i(Y) \leq f_i(X)$ for $i = 1, \dots, k$ and $f_j(Y) < f_j(X)$ for at least one objective function. This means that a feasible vector X is called Pareto optimal if there is no other feasible solution Y that would reduce some objective function without causing a simultaneous increase in at least one other objective function. The set of all feasible non-dominated solutions in X is referred to as the

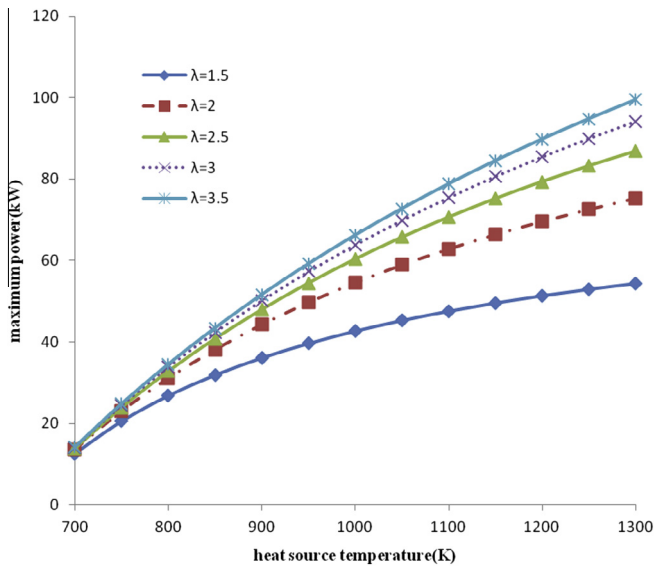


Fig. 6. Variation of the power of the Stirling engine for different the heat source temperature and λ .

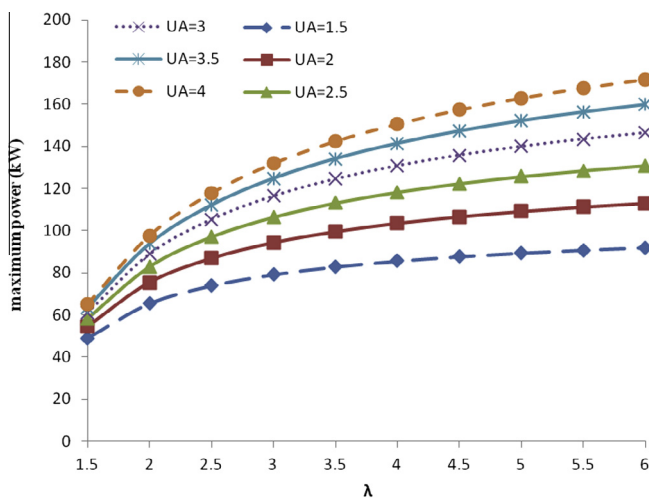


Fig. 7. Variation of the power of the Stirling engine for different λ and UA.

“Pareto optimal set”, and for a given Pareto optimal set, the corresponding objective function values in the objective space are called the “Pareto optimal frontier” [48].

For having a helpful insight into the concept of Pareto optimal consider Fig. 12. This figure demonstrates the objective functions space for an optimization problem with two objective functions f_1 and f_2 . The feasible and infeasible areas are demonstrated as the areas inside and outside of the curve respectively. Every point in the feasible area is a solution of the problem. In Fig. 12, the values of both functions f_1 and f_2 , for point M are lower than the corresponding values of point j. Thus, point M dominates point j or point M is better than point j. In the same way, points L and N dominate M. But points like I and K neither dominate M nor are dominated by M. Therefore, only those points that are located in the left-down parts of M will dominate M (like L and N). Now consider point R, there is no point in the feasible area located in the left-down part of it. Thus, R is not dominated by any other feasible point. Therefore, R is a Pareto optimal. This is true for points P, A, R, E, T, O and all of the other points located at the bold curve indicated as “The Pareto Optimal Frontier” in Fig. 12. Notice that, in the feasible area, the minimum values of f_1 and f_2 belong to

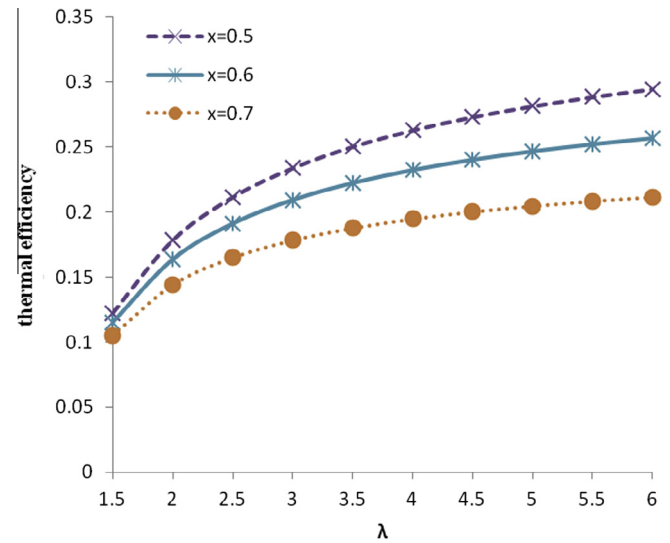


Fig. 8. Variation of the thermal efficiency of the Stirling engine for different λ and x .

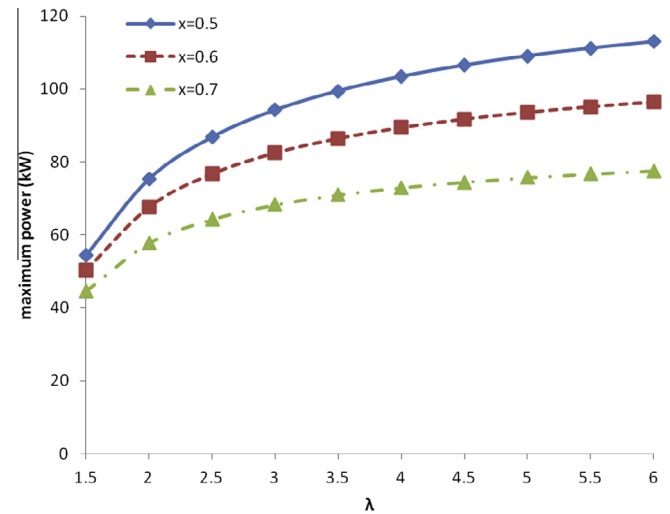


Fig. 9. Variation of the maximum power of the Stirling engine for different λ and x .

points P and O, respectively. Thus, P and O are the solutions for uni-objective optimization problems which their objective functions are f_1 and f_2 , respectively. Other points on the Pareto frontier are also optimal, but which of them should be chosen as the final solution. For doing this a decision-making process is required.

2.2.2. Optimization via EA

In the present study, the Pareto frontier is achieved using the Genetic Algorithm (GA) which is a branch of evolutionary algorithm. Genetic Algorithms were evolved by John Holland in the 1960s to import the mechanisms of natural adaptation into computer algorithms and numerical optimization [50]. They are used as a computer simulation in a way that a population of abstract representations (called chromosomes or the genotype of the genome) of candidate solutions (called individuals, creatures, or phenotypes) to an optimization problem develops toward better solutions. It begins from a population of randomly produced exclusives and occurs in generations. In every generation, the fitness of each individual is measured; multiple individuals are stochastically chosen from the current population, and altered to form a

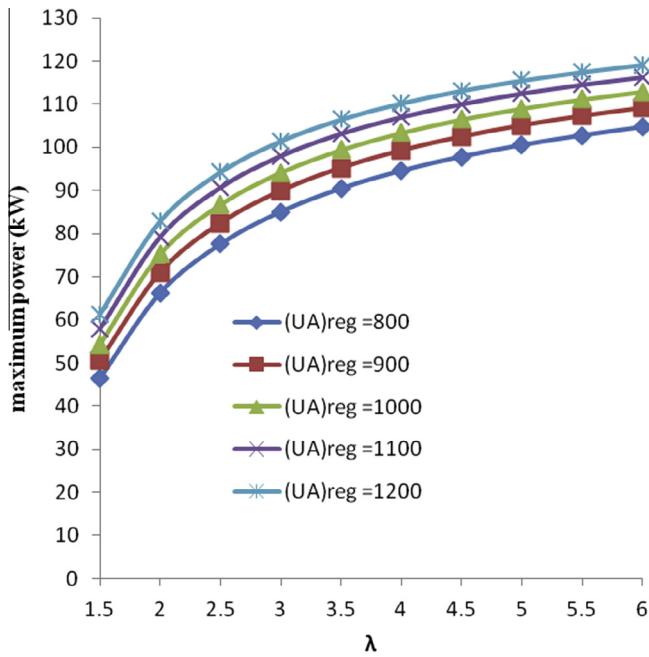


Fig. 10. Variation of the maximum power of the Stirling engine for different λ and $(UA)_{reg}$.

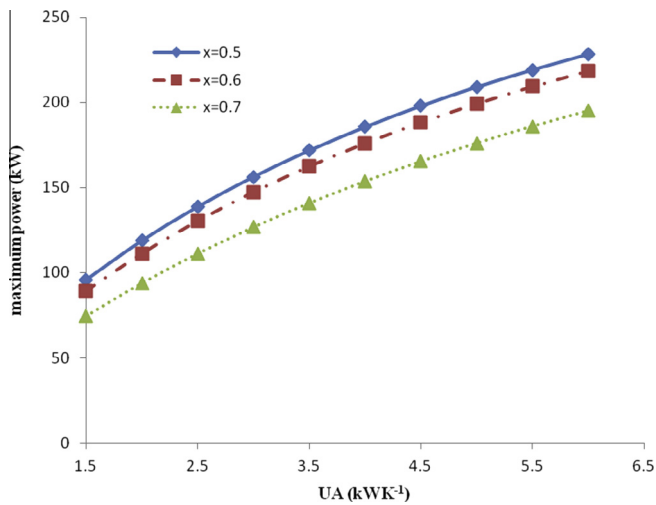


Fig. 11. Variation of the power of the Stirling engine for different UA and x .

new population. The new group is employed in the next iteration. Generally, the algorithm finishes when either a maximum number of generations have been produced, or a satisfactory fitness level has been gained for the population. If the algorithm has ended owing to a maximum number of generations, a satisfactory solution might have been reached or might not have been accomplished. In genetic algorithms, a candidate solution to a problem is named a chromosome, and the evolutionary viability of each chromosome is shown by a fitness function. This method can be applied to optimize nonlinear problems [48,51].

Multi-objective evolutionary algorithms (MOEAs) have been evolved in the recent decades by a lot of tests on complex mathematical problems. On real-world it has shown that they can omit the obstacles of classical methods [48,51]. The structure of the MOEA exploited in this investigation is depicted in Fig. 13 [50]. The real values of decision variables are applied instead of their binary coded.

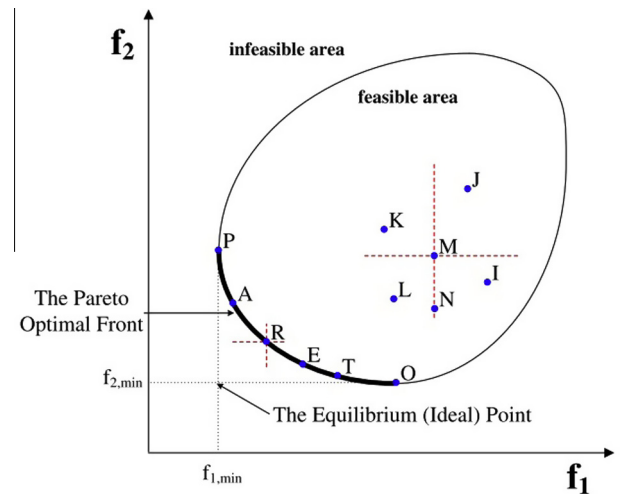


Fig. 12. Schematic of the objectives space [35].

2.2.3. Objective functions, decision variables and constraints

Two objective functions for this study are the system's output power (P) and the Stirling engine's thermal efficiency (η_t) denoted by Eqs. (9) and (12), respectively.

In this paper six decision variables have been considered as follow:

T_H	Temperature of heat source
A_R	Heat transfer area ratio
ψ	$\frac{U_L}{U_H}$
T_h	Temperatures of the working fluid in the high temperature isothermal process 3–4
x	Temperature ratio
$U_H A_H$	Heat transfer coefficient between the engine and the high-temperature source

The objective functions with respect to following constraints have been solved:

$$1100 \leq T_H \leq 1300 \text{ K} \quad (16)$$

$$800 \leq T_h \leq 1000 \text{ K} \quad (17)$$

$$0.5 \leq \psi \leq 1 \quad (18)$$

$$0.5 \leq A_R \leq 1 \quad (19)$$

$$0.5 \leq x \leq 0.7 \quad (20)$$

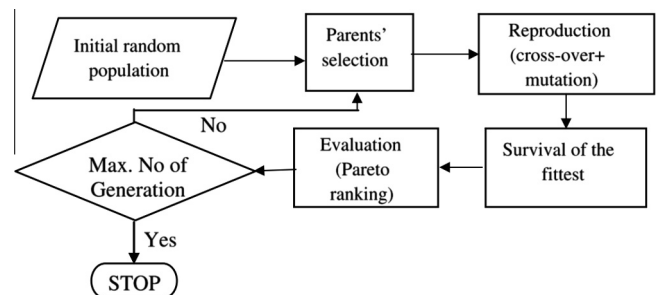


Fig. 13. Scheme for the multi-objective evolutionary algorithm used in the present study [34–36].

$$1.5 \leq U_H A_H \leq 6.5 \text{ kW K}^{-1} \quad (21)$$

2.3. Decision-making in the multi-objective optimization

In multi-objective optimization, a procedure of decision-making for opting the final optimal solution is implicated. There are a lot of decision-making methods and these methods can be utilized in decision-making to choose the final optimal solution from the Pareto frontier which is achieved by optimization. Owing to different dimensions of various objectives in a multi-objective optimization problem and necessity of unified dimensions and scales of objectives space, in turn, objectives vectors should be non-dimensionalized before the decision-making process. Some methods such as linear, Euclidian, and fuzzy can be utilized for non-dimensionalization.

• Linear non-dimensionalization

Consider the matrix of objectives at various points of the Pareto frontier is denoted by F_{ij} where i is index for each point on the Pareto frontier and j is the index for each objective in objectives space. Therefore, a non-dimensionalized objective, F_{ij}^n , is defined as,

$$F_{ij}^n = \frac{F_{ij}}{\max(F_{ij})} \text{ for maximizing objective} \quad (22a)$$

$$F_{ij}^n = \frac{F_{ij}}{\max(1/F_{ij})} \text{ for minimalizing objective} \quad (22b)$$

• Euclidian non-dimensionalization

In this method, a non-dimensionalized objective, F_{ij}^n , is defined as,

$$F_{ij}^n = \frac{F_{ij}}{\sqrt{\sum_{i=1}^m (F_{ij})^2}} \text{ for minimizing and maximizing objectives} \quad (23)$$

• Fuzzy non-dimensionalization

In this method, a non-dimensionalized objective, F_{ij}^n , is defined as,

$$F_{ij}^n = \frac{F_{ij} - \min(F_{ij})}{\max(F_{ij}) - \min(F_{ij})} \text{ for maximizing objectives} \quad (24a)$$

$$F_{ij}^n = \frac{\max(F_{ij}) - F_{ij}}{\max(F_{ij}) - \min(F_{ij})} \text{ for minimizing objectives} \quad (24b)$$

In the present research, most well-known kind of decision-making procedures such as the fuzzy Bellman–Zadeh, LINMAP and TOPSIS method are employed in parallel and the final optimal solution has been chosen based on real-world experience and criteria among the suggested solutions. The Bellman–Zadeh method uses the fuzzy non-dimensionalization while the other methods (LINMAP and TOPSIS) utilize Euclidian non-dimensionalization. The mentioned decision-making algorithms are represented in the following.

2.3.1. Bellman–Zadeh decision-making method

When using the Bellman–Zadeh approach, each $F_j(X)$ is replaced by a fuzzy objective function or a fuzzy set,

$$A_j = \{X, \mu_{A_j}(X)\} \quad X \in L, j = 1, 2, \dots, k \quad (25)$$

where $\mu_{A_j}(X)$ is a membership function of A_j [57].

A final decision is defined by the Bellman and Zadeh model as the intersection of all fuzzy criteria and constraints and is represented by its membership function. A fuzzy solution D with setting

up the fuzzy sets (Eq. (26)) is turned out as a result of the intersection $D = \cap_{j=1}^k A_j$ with a membership function,

$$\mu_D(X) = \cap_{j=1}^k \mu_{A_j}(X) = \min_{j=1, \dots, k} \mu_{A_j}(X) \quad X \in L \quad (26)$$

Using Eq. (24a), it is possible to obtain the solution proving the maximum degree as follows,

$$\max \mu_D(X) = \max_{X \in L} \min_{j=1, \dots, k} \mu_{A_j}(X) \quad (27)$$

$$X^0 = \arg \max_{X \in L} \min_{j=1, \dots, k} \mu_{A_j}(X) \quad (28)$$

To obtain Eq. (27), it is necessary to build membership functions $\mu_{A_j}(X); j = 1, \dots, k$, reflecting a degree of achieving “own” optima by the corresponding $F_j(X), X \in L; j = 1, \dots, k$. This is satisfied by the use of the membership functions [58]. The membership function of objectives and constraints, linear or nonlinear, can be chosen depending on the context of problem. One of possible fuzzy convolution schemes is presented below [59].

Initial approximation for X -vector is chosen. Maximum (minimum) values for each criterion $F_j(X)$ are established via scalar maximization (minimization). Results are denoted as “ideal” points $\{X_j^0, j = 1, \dots, m\}$.

The matrix table $\{T\}$, where the diagonal elements are “ideal” points, is defined as follows:

$$\{T\} = \begin{bmatrix} F_1(X_1^0) & F_2(X_1^0) & \dots & F_n(X_1^0) \\ F_1(X_2^0) & F_2(X_2^0) & \dots & F_n(X_2^0) \\ \vdots & \vdots & \ddots & \vdots \\ F_1(X_n^0) & F_2(X_n^0) & \dots & F_n(X_n^0) \end{bmatrix} \quad (29)$$

Maximum and minimum bounds for the criteria are defined:

$$F_i^{\min} = \min_j F_j(X_j^0) \quad i = 1, \dots, n \quad (30a)$$

$$F_i^{\max} = \max_j F_j(X_j^0) \quad i = 1, \dots, n \quad (30b)$$

The membership functions are assumed for all fuzzy goals as follows. For minimized objective functions:

$$\mu_{F_i}(X) = \begin{cases} 0 & \text{if } F_i(X) > F_i^{\max} \\ \frac{F_i^{\max} - F_i}{F_i^{\max} - F_i^{\min}} & \text{if } F_i^{\min} < F_i \leq F_i^{\max} \\ 1 & \text{if } F_i(X) \leq F_i^{\min} \end{cases} \quad (31a)$$

For maximized objective functions:

$$\mu_{F_i}(X) = \begin{cases} 1 & \text{if } F_i(X) > F_i^{\max} \\ \frac{F_i - F_i^{\min}}{F_i^{\max} - F_i^{\min}} & \text{if } F_i^{\min} < F_i \leq F_i^{\max} \\ 0 & \text{if } F_i(X) \leq F_i^{\min} \end{cases} \quad (31b)$$

Fuzzy constraints are formulated:

$$G_j(X) \leq G_j^{\max} + d_j, \quad j = 1, 2, \dots, k \quad (32)$$

where d_j is a subjective parameter that denotes a distance of admissible displacement for the bound $G_j^{\max}$ of the j th constraint? Corresponding membership functions are defined in following manner:

$$\mu_{G_i}(X) = \begin{cases} 0 & \text{if } G_i(X) > G_i^{\max} \\ 1 - \frac{G_j(X) - G_j^{\max}}{d_j} & \text{if } G_i^{\max} < G_i(X) \leq G_i^{\max} + d_j \\ 1 & \text{if } G_i(X) \leq G_i^{\max} \end{cases} \quad (33)$$

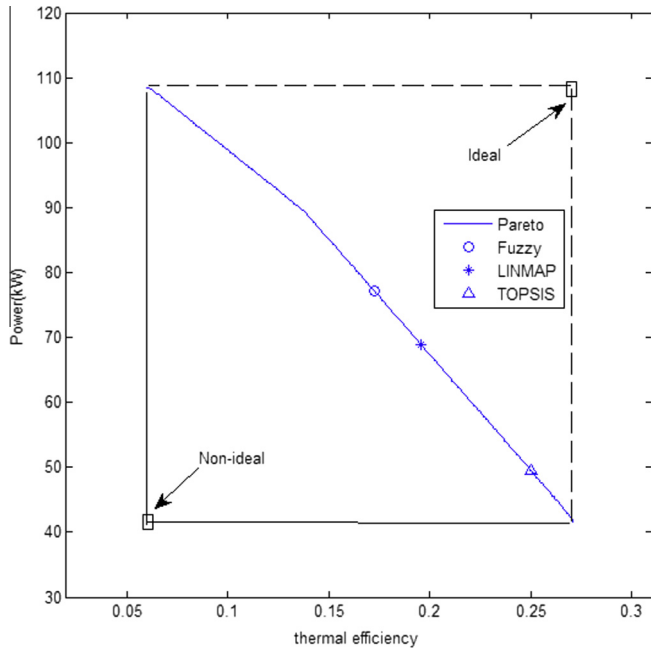


Fig. 14. Pareto optimal frontier in objectives' space.

A final decision is determined as the intersection of all fuzzy criteria and constraints represented by its membership functions. This problem is reduced to the standard nonlinear programming problems: to find such values of X and k that maximize k subject to

$$\begin{aligned} \lambda &\leq \mu_{F_i}, \quad i = 1, 2, \dots, n \\ \lambda &\leq \mu_{G_j}, \quad j = 1, 2, \dots, k \end{aligned} \quad (34)$$

The solution of the multi-criteria problem discloses the meaning of the optimality operator and depends on the decision-makers experience and problem understanding.

2.3.2. LINMAP decision-making method

An ideal point on the Pareto frontier is the point in which each objective is optimized regardless to satisfaction of other objectives. It is clear that in the multi-objective optimization it is impossible to have each objective in its optimal condition that it can obtain in a single-objective optimization. Therefore, the ideal point is not located on the Pareto frontier. In the LINMAP method, after Euclidian non-dimensionalization of all objectives, the distance of each solution on the Pareto frontier from the ideal point denoted by d_{i+} is determined as follow,

$$d_{i+} = \sqrt{\sum_{j=1}^n F_{ij} - F_j^{ideal}} \quad (35)$$

where denotes the number of objective while stand for each solution on the Pareto frontier ($i = 1, 2, \dots, m$). In Eq. (35), F_j^{ideal} is the ideal value for j th objective obtained in a single-objective optimization. In LINMAP method, the solution with minimum distance from

ideal point is selected as a final desired optimal solution, hence, i index for a final solution, i_{final} is,

$$i_{final} = i \in \min(d_{i+}); \quad i = 1, 2, \dots, m \quad (36)$$

2.3.3. TOPSIS decision-making method

In this method, beside the ideal point a non-ideal point is defined. The non-ideal point is the ordinate in objectives space in which each objective has its worst value. Therefore, beside the solution distance from ideal point, d_{i+} , the solution distance from non-ideal point denoted by d_{i-} is used as a criterion for selection of the final solution. Hence,

$$d_{i-} = \sqrt{\sum (F_{ij} - F_j^{non-ideal})^2} \quad (37)$$

In continuing the TOPSIS method a parameter is defined as follows,

$$Cl_i = \frac{d_{i-}}{d_{i+} + d_{i-}} \quad (38)$$

In TOPSIS method, a solution with minimum Cl_i is selected as a desired final solution, therefore, if i_{final} is index for the final selected solution, we have,

$$i_{final} = i \in \max(Cl_i); \quad i = 1, 2, \dots, m \quad (39)$$

2.4. Second scenario results

The system's output power and thermal efficiency of the Stirling engine are maximized simultaneously using the multi-objective optimizing method which works based on the NSGA-II algorithm.

In this regard, optimization is performed with objective functions that are expressed by Eqs. (9) and (12) constraints which are expressed with Eqs. (16)–(21). Design parameters (decision variables) of optimization are the temperature of heat source, heat transfer area ratio, ψ , temperatures of the working fluid in the high temperature isothermal process 3–4, temperature ratio, Heat transfer coefficient between the engine and the high-temperature source.

Fig. 14 illustrates the Pareto frontier in the proposed objectives' space obtained using the multi-objective evolutionary algorithm (NSGA-II). Three final solutions selected by the Bellman–Zadeh, LINMAP and TOPSIS decision-makers are indicated in Fig. 14.

The results of two-objective optimization based on Fuzzy, LINMAP and TOPSIS decision making methods are shown in Table 1. In this Table, results for P and η_t objective functions based on six decision variables are categorized. Temperature of heat source (T_H) is obtained between 1100 K which is unacceptable value due to previous studies [42]. Temperature ratio (x) is obtained between 0.500 and 0.501 which is unacceptable range due to previous studies [42].

Table 2 compares optimal results obtained in this paper by two scenarios.

Table 1
Decision making of multi-objective optimal solutions.

Decision making method	Decision variables						Objective functions	
	T_H (K)	T_h (K)	ψ	A_R	x	UA (kW K ⁻¹)	P (kW)	η_t (%)
Bellman–Zadeh	1100.7	999.75	0.996	0.996	0.501	4.427	77.114	17.25
LINMAP	1100.6	999.8	0.998	0.997	0.501	3.485	68.874	19.6
TOPSIS	1100.6	999.8	0.998	0.996	0.500	1.961	49.510	25.04

Table 2

Values of variables and objective functions obtained in two optimization scenarios.

	Decision making method	Variables						Objective functions	
		T_H (K)	T_c (K)	ψ	A_R	x	UA (kW K ⁻¹)	P (kW)	η_c (%)
Scenario 1	–	1300	970	1.000	1.000	0.500	2.000	75.306	17.82
Scenario 2	Bellman–Zadeh	1100.7	999.75	0.996	0.996	0.501	4.427	77.114	17.25
	LINMAP	1100.6	999.8	0.998	0.997	0.501	3.485	68.874	19.6
	TOPSIS	1100.6	999.8	0.998	0.996	0.500	1.961	49.510	25.04

3. Conclusions

In the presented paper, the effects of some parameters such as UA , $(UA)_{reg}$, heat source temperature (T_H) and volumetric ratio (λ) on the output power and the thermal efficiency is investigated. It can be concluded that the effects of UA is more than $(UA)_{reg}$ on the output power. The temperature of heat source is one of the important parameters affecting the output power and by increasing of temperature the power increases too. This behavior varies with changes in other parameters.

The volumetric ratio has an effective role in output power and by increasing it, the power increases, but the slope of the curve decreases by increasing temperature ratio; Also, the effects of λ in the range 1.5–3.5 is sensible. Volumetric ratio affects thermal efficiency similar to the output power. Also, the impacts of x on the power and thermal efficiency are evaluated. The results of two scenarios show that the theory can be useful for design and the performance analysis of Stirling heat engine.

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